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CROSS RATIOS BETWEEN DALITZ PLOT AMPLITUDES IN THREE-BODY D^0 DECAYS

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A recent study of $D^0 \to \pi^0 K^+ K^-$ and $D^0 \to K_S \pi^+ \pi^-$ describes a flavor-symmetric approach to calculate relative amplitudes and phases, for characteristic interferences between D decays to a light pseudoscalar P and a light vector V, on Dalitz plots for $D \to PPP$ decays. The flavor-symmetric approach used an earlier fit to $D \to PV$ decay rates and was found to agree fairly well with experiments for $D^0 \to \pi^0 \pi^+ \pi^-$ but not as well for $D^0 \to \pi^0 K^+ K^-$ and $D^0 \to K_S \pi^+ \pi^-$. The present work extends this investigation to include $D^0 \to K^- \pi^+ \pi^0$. We use an SU(3) flavor symmetry relationship between ratios of Cabibbo-favored (CF) $D \to PV$ amplitudes in $D^0 \to K^- \pi^+ \pi^0$ and ratios of singly- Cabibbo-suppressed (SCS) $D \to PV$ amplitudes in $D^0 \to \pi^0 K^+ K^-$ and $D^0 \to \pi^0 \pi^+ \pi^-$. We observe that experimental values for Dalitz plot cross ratios obey this relationship up to discrepancies noted previously. The need for an updated Dalitz plot analysis of $D^0 \to K^- \pi^+ \pi^0$ is emphasized.

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I Introduction

Decays of D mesons to a light pseudoscalar meson P and a light vector meson V were studied earlier in Refs. [1, 2] using SU(3) flavor-symmetry. The results of these analyses were applied to extract relative phases and amplitudes for quasi-two-body (PV) final states in three-body Dalitz plots. Agreement with experiment [3, 4, 5] was found to be good for the $D^0 \to \pi^0 \pi^+ \pi^-$ Dalitz plot [6], but poorer [7] for $D^0 \to \pi^0 K^+ K^-$ and $D^0 \to K_S \pi^+ \pi^-$ [8, 9, 10, 11, 12, 13].

In this paper we revisit the relative phases and amplitudes of characteristic interferences of $D \to PV$ decays on $D^0 \to PPP$ Dalitz plots. We compare the Dalitz plot analyses of $D^0 \to \pi^0 K^+ K^-$ and $D^0 \to \pi^0 \pi^+ \pi^-$ which were previously studied in two different contexts. We also include a comparison of these Dalitz plots with the Dalitz plot for $D^0 \to K^- \pi^+ \pi^0$. We compare ratios of $D \to PV$ amplitudes obtained from each Dalitz plot analysis with predictions from the flavor-symmetric technique, and find a fair match with experimental data. This agreement is useful in validating both the flavor-symmetric technique and the sign conventions used in each Dalitz plot analysis.

In Sec. II we recall our notation for the SU(3) flavor-symmetric analysis and quote the values of the relevant parameters obtained in earlier fits [1, 2]. In Sec. III we construct the $D \to PV$ amplitudes that are relevant for our present study. Sec. IV compares ratios of $D \to PV$ amplitudes obtained using Dalitz plot fit fractions with the predictions of the

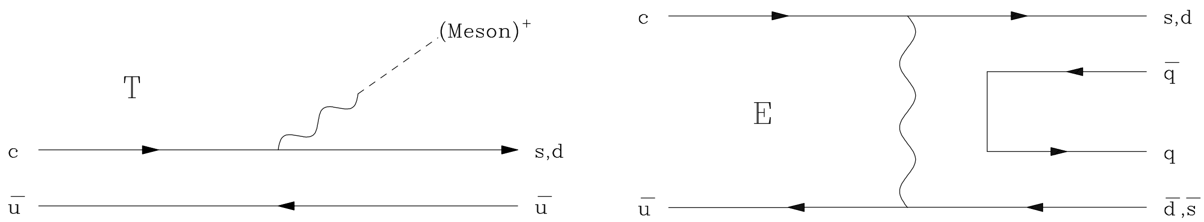

Figure 1: Flavor topologies for describing charm decays. T: color-favored tree; E exchange. Not shown: C (color-suppressed tree); A (annihilation).

flavor-symmetric analysis. We compare our results with those of previous analyses in Sec. V and conclude in Sec. VI.

II Amplitudes from previous fits

The notation for the SU(3) flavor-symmetric analysis of $D \to PV$ decays is discussed in Ref. [1]. Here we briefly recall some of the salient features. We denote Cabibbo-favored (CF) amplitudes, proportional to the product $V_{ud}V^*_{cs}$ of Cabibbo-Kobayashi-Maskawa (CKM) factors, by unprimed amplitudes. The singly-Cabibbo-suppressed (SCS) amplitudes, proportional to the product $V_{us}V^*_{cs}$ or $V_{ud}V^*_{cd}$, are then obtained by using the ratio SCS/CF= $\tan\theta_C \equiv \lambda = 0.2305$ [14], with θ_C the Cabibbo angle and signs governed by the relevant CKM factors.

The present scenario involves the amplitudes labeled as T ("tree") and E ("exchange"), illustrated in Fig. 1. The subscript P or V on an amplitude denotes the meson (P or V) containing the spectator quark in the PV final state. The partial width $\Gamma(H \to PV)$ for the decay of a heavy meson H is given in terms of an invariant amplitude $\mathcal{A}$ as

$$\Gamma(H \to PV) = \frac{p^{*3}}{8\pi M_H^2}|\mathcal{A}|^2, \tag{1}$$

where p^* is the center-of-mass (c.m.) 3-momentum of each final particle, and M_H is the mass of the decaying heavy meson. With this definition the amplitudes $\mathcal{A}$ are dimensionless.

The amplitudes T_V and E_P were obtained from fits to rates of CF $D \to PV$ decays not involving η or η' [1]. To specify the amplitudes T_P and E_V, however, one needs information on the η–η' mixing angle (θ_η). Table I summarizes these results for two values $\theta_\eta = 19.5^\circ$ and 11.7°. In order to make the discussion complete we also quote the results for C_P and C_V in Table I. As described in [1] the amplitudes and phases for these parameters were also obtained from fits to rates of CF $D \to PV$ decays.

III $D \to PV$ amplitudes in the flavor-symmetric approach

We list the $D^0 \to PV$ amplitudes appearing in Dalitz plots of interest for the present discussion in Tables II (for $\theta_\eta = 19.5^\circ$) and III (for $\theta_\eta = 11.7^\circ$), including their representations and values in terms of flavor SU(3) amplitudes.

Table I: Solutions for T_V, E_P, C_P, T_P, E_V and C_P amplitudes in Cabibbo-favored charmed meson decays to PV final states, for η–η' mixing angles of $\theta_\eta = 19.5°$ and $11.7°$.

	$\theta_\eta = 19.5°$		$\theta_\eta = 11.7°$	
PV ampl.	Magnitude (10^{-6})	Relative strong phase	Magnitude (10^{-6})	Relative strong phase
T_V	3.95±0.07	–	These results are independent of θ_η	
E_P	2.94±0.09	$\delta_{E_P T_V} = (-93 \pm 3)°$		
C_P	4.88±0.15	$\delta_{C_P T_V} = (-162 \pm 1)°$		
T_P	7.46±0.21	Assumed 0	7.69±0.21	Assumed 0
E_V	2.37±0.19	$\delta_{E_V T_V} = (-110 \pm 4)°$	1.11±0.22	$\delta_{E_V T_V} = (-130 \pm 10)°$
C_V	3.46±0.18	$\delta_{C_V T_V} = (172 \pm 3)°$	4.05±0.17	$\delta_{C_V T_V} = (162 \pm 4)°$

Table II: Amplitudes for $D^0 \to PV$ decays corresponding to Dalitz plots of interest for the present discussion (in units of 10^{-6}). Here we have taken $\theta_\eta = 19.5°$.

Dalitz plot	D^0 final state	Amplitude representation	Amplitude A Re	Im	$\|A\|$	Phase (°)
	$\rho^+\pi^-$	$-\lambda(T_P + E_V)$	–1.533	0.513	1.616	161.5
$D^0 \to \pi^0\pi^+\pi^-$	$\rho^-\pi^+$	$-\lambda(T_V + E_P)$	–0.875	0.677	1.106	142.3
	$\rho^0\pi^0$	$\frac{\lambda}{2}(E_P + E_V - C_P - C_V)$	0.819	–0.477	0.947	-30.2
	$K^{*+}K^-$	$\lambda(T_P + E_V)$	1.533	–0.513	1.616	–18.5
$D^0 \to \pi^0 K^+ K^-$	$K^{*-}K^+$	$\lambda(T_V + E_P)$	0.875	–0.677	1.106	–37.7
	$\phi\pi^0$	$\lambda\, C_P/\sqrt{2}$	–0.756	–0.246	0.795	–162
	ρ^+K^-	$T_P + E_V$	6.649	–2.227	7.012	–18.5
$D^0 \to K^-\pi^+\pi^0$	$K^{*-}\pi^+$	$T_V + E_P$	3.796	–2.936	4.799	–37.7
	$\overline{K}^{*0}\pi^0$	$(C_P - E_P)/\sqrt{2}$	–3.173	1.010	3.33	162.3

The amplitude representations in these tables imply the following interesting relationships between the amplitudes:

$$\mathcal{A}(D^0 \to \rho^+\pi^-) = -\mathcal{A}(D^0 \to K^{*+}K^-) = \lambda\mathcal{A}(D^0 \to \rho^+K^-) \quad (2)$$

$$\mathcal{A}(D^0 \to \rho^-\pi^+) = -\mathcal{A}(D^0 \to K^{*-}K^+) = \lambda\mathcal{A}(D^0 \to K^{*-}\pi^+) \quad (3)$$

The above relationships are based only on SU(3) flavor symmetry and imply a relative phase of 0° or 180° between amplitudes, independent of the fitted parameter values. Taking ratios of these we obtain a ratio with the same magnitude and phase independent of which Dalitz plot analysis one chooses to look at:

$$R\, e^{i\Phi} = \frac{\mathcal{A}(D^0 \to \rho^+\pi^-)}{\mathcal{A}(D^0 \to \rho^-\pi^+)} = \frac{\mathcal{A}(D^0 \to K^{*+}K^-)}{\mathcal{A}(D^0 \to K^{*-}K^+)} = \frac{\mathcal{A}(D^0 \to \rho^+K^-)}{\mathcal{A}(D^0 \to K^{*-}\pi^+)}, \quad (4)$$

where R and Φ, respectively, denote the magnitude and phase of the ratio. In the following section we calculate this ratio using Dalitz plot fits and compare it with the predictions from the SU(3)-flavor-symmetric approach.

Table III: Same as Table II except with $\theta_\eta = 11.7°$.

Dalitz plot	D^0 final state	Amplitude representation	Amplitude A Re	Im	$\lvert A\rvert$	Phase (°)
	$\rho^+\pi^-$	$-\lambda(T_P + E_V)$	–1.608	0.196	1.620	173.1
$D^0 \to \pi^0\pi^+\pi^-$	$\rho^-\pi^+$	$-\lambda(T_V + E_P)$	–0.875	0.677	1.106	142.3
	$\rho^0\pi^0$	$\frac{\lambda}{2}(E_P + E_V - C_P - C_V)$	0.879	–0.407	0.968	–24.8
	$K^{*+}K^-$	$\lambda(T_P + E_V)$	1.608	–0.196	1.620	–6.9
$D^0 \to \pi^0 K^+K^-$	$K^{*-}K^+$	$\lambda(T_V + E_P)$	0.875	–0.677	1.106	–37.7
	$\phi\pi^0$	$\lambda\, C_P/\sqrt{2}$	–0.756	–0.246	0.795	–162
	ρ^+K^-	$T_P + E_V$	6.977	–0.850	7.028	–6.9
$D^0 \to K^-\pi^+\pi^0$	$K^{*-}\pi^+$	$T_V + E_P$	3.796	–2.936	4.799	–37.7
	$\overline{K}^{*0}\pi^0$	$(C_P - E_P)/\sqrt{2}$	–3.173	1.010	3.33	162.3

IV Comparison of Dalitz plot data from experiments with predictions

The representations of $D^0 \to PV$ amplitudes mentioned in the previous section do not contain the information about the vector meson decay to a pair of light pseudoscalars. The fit fractions for an intermediate process $D^0 \to RC$ in a $D^0 \to ABC$ Dalitz plot (A, B and C are light pseudoscalars; R is the intermediate light vector resonance AB), however, also include the fraction of the process $R \to AB$. The fraction of the vector meson's decay to a pair of pseudoscalars can be simply represented by the corresponding isospin Clebsch-Gordan factor.

It is important to note that the spin part of the amplitude for the process $D^0 \to RC \to ABC$ is given by $T = -2\vec{p}_A \cdot \vec{p}_C$ where $\vec{p}_i$ is the 3-momentum of the particle i in the resonance rest frame, implying that the phase of the corresponding amplitude changes by π if we switch the order of the daughters in the vector decay. The conventions used in the present analysis are the same as those previously used for $D^0 \to \pi^0\pi^+\pi^-$ in Ref. [6, 15, 16], for $D^0 \to \pi^0K^+K^-$ in Ref. [7, 15, 16], and for $D^0 \to K^-\pi^+\pi^0$ in Eq. (12) of Ref. [17]. The relevant Clebsch-Gordan factors are noted alongside the respective index conventions in Table IV where we use Ref. [18] for appropriate sign conventions.

Making use of the appropriate isospin Clebsch-Gordan factors listed in Table IV, we may now calculate the magnitudes and phases of the amplitudes listed in Eqs. (3) and (4). This is done in Table V for $\theta_\eta = 19.5°$, where we also present the corresponding results from SU(3) flavor-symmetry, side-by-side for easy comparison. In Table V we also quote the magnitudes and phases of other amplitudes from the relevant Dalitz plots for the sake of completeness. In order to obtain the experimental amplitudes to compare with theory, we use the corresponding fit fractions from Refs. [5, 13, 17] and normalize them so as to set the larger of the two amplitudes to equal 1. In case the two processes involved in this normalization have different values for the momentum p^* then we also include a factor of the ratio p_A^{*3}/p_B^{*3} where A and B are the two processes involved in appropriate order. This takes account of unequal phase space factors between the two processes. For the $D^0 \to K^-\pi^+\pi^0$ Dalitz plot we use the data from both the "3-resonance fit" and the "final

Table IV: Conventions for order of the two pseudoscalars in vector meson decay [6, 7, 17].

Dalitz Plot	Bachelor Particle		Vector Meson Decay		
	Meson	Index	Process	Indices	Clebsch factor
	π^0	1	$\rho^0 \to \pi^+\pi^-$	23	1
$D^0 \to \pi^0\pi^+\pi^-$	π^+	2	$\rho^- \to \pi^0\pi^-$	13	1
	π^-	3	$\rho^+ \to \pi^0\pi^+$	12	-1
	π^0	1	$\phi \to K^+K^-$	23	$1/\sqrt{2}$
$D^0 \to \pi^0 K^+K^-$	K^+	2	$K^{*-} \to K^-\pi^0$	31	$-1/\sqrt{3}$
	K^-	3	$K^{*+} \to \pi^0 K^+$	12	$-1/\sqrt{3}$
	K^-	1	$\rho^+ \to \pi^+\pi^0$	23	1
$D^0 \to K^-\pi^+\pi^0$	π^+	2	$K^{*-} \to K^-\pi^0$	13	$-1/\sqrt{3}$
	π^0	3	$\overline{K}^{*0} \to K^-\pi^+$	12	$-\sqrt{2/3}$

Table V: Amplitudes for $D^0 \to PV$ decays from Dalitz plots of interest for the present discussion (in units of 10^{-6}). Here we have taken $\theta_\eta = 19.5^\circ$, and $\lambda = 0.2305$ [14]. The experimental amplitudes have arbitrary overall normalization.

D^0 final state	Amplitude representation	Theory [1, 6, 7] Amplitude	Phase (°)	Experiment * Clebsch factor Amplitude	Rel. Phase (°)
$A(\rho^+\pi^-)$	$-\lambda(T_P + E_V)$	1.616±0.060	161.5±1.6	0.823±0.004	0(def.) [5]
$A(\rho^-\pi^+)$	$-\lambda(T_V + E_P)$	1.106±0.033	142.3±1.5	0.588±0.008	– 2±0.6±0.6
$A(\rho^0\pi^0)$	$\frac{\lambda}{2}(E_P + E_V$ $-C_P - C_V)$	0.947±0.036	–30.2±2.1	0.512±0.012	–163.8±0.6±0.4
$A(K^{*+}K^-)$	$\lambda(T_P + E_V)$	1.616±0.060	–18.5±1.6	1(def.)	0(def.) [13]
$A(K^{*-}K^+)$	$\lambda(T_V + E_P)$	1.106±0.033	–37.7±1.5	0.601±0.016	–37.0±1.9±2.2
$A(\phi\pi^0)$	$\lambda\, C_P/\sqrt{2}$	0.795±0.023	–162±1	0.69±0.02	159.3±13.6±9.3
$\lambda A(\rho^+K^-)$	$\lambda(T_P + E_V)$	1.616±0.060	–18.5±1.6	1(def.)	0(def.) [17]
$\lambda A(K^{*-}\pi^+)$	$\lambda(T_V + E_P)$	1.106±0.033	–37.7±1.5	0.631±0.015^a	–13.3±2.0^a
				0.725±0.050^b	–17.0±5.8^b
$\lambda A(\overline{K}^{*0}\pi^0)$	$\frac{\lambda}{\sqrt{2}}(C_P - E_P)/$	0.768±0.033	162.3±1.7	0.457±0.036^a	172.8±2.2^a
				0.494±0.011^b	179.8±8.0^b

^{a} Data from “3-resonance fit” of Ref. [17].
^{b} Data from “final fit” of Ref. [17].

Table VI: Comparison between predicted and measured ratios in Eq. (4). Inputs were taken from Table V above.

Amplitude	Predicted		Measured	
ratio	Magnitude (R)	Phase (Φ°)	Magnitude	Phase (Φ°)
$A(\rho^+\pi^-)/A(\rho^-\pi^+)$	1.461±0.070	19.2±2.2	1.400±0.020	2.0±0.8
$A(K^{*+}K^-)/A(K^{*-}K^+)$	1.461±0.070	19.2±2.2	1.664±0.043	37.0±2.9
$A(\rho^+K^-)/A(K^{*-}\pi^+)$	1.461±0.070	19.2±2.2	1.585±0.037[a]	13.3±2.0[a]
			1.379±0.100[b]	17.0±5.8[b]

[a] Data from "3-resonance fit" of Ref. [17].
[b] Data from "final fit" of Ref. [17].

fit" mentioned in Ref. [17].

In Table VI we compare the magnitude R and the phase Φ of the ratio in Eq. (4) obtained from the three different Dalitz plots with the predictions from the SU(3) flavor-symmetric analysis. It is important to notice that the agreement between theory and experiment on the value of R is expected since the SU(3) flavor-symmetric approach makes use of some these experiments. The fact that the corresponding relative phases agree fairly well with each other, however, is working evidence for the flavor-symmetric approach. A similar exercise when performed for $\theta_\eta = 11.7^\circ$ does not give any significant changes and hence is omitted from this discussion.

V Comparison with previous analyses

The results quoted in Tables V and VI are worth comparing with our earlier results quoted in Refs. [6, 7]. The first three rows in Table V involve amplitudes and phases from the $D^0 \to \pi^0\pi^+\pi^-$ Dalitz plot. These results agree with the findings of Ref. [6]. The relative amplitudes obtained using flavor SU(3) compare very well with those found from experiment. There is, however, some discrepancy between the relative phases obtained experimentally and from theory. If we set the phase for $A(\rho^+\pi^-)$ to zero, then we obtain a relative phase discrepancy of about 17° for $A(\rho^-\pi^+)$ and 28° for $A(\rho^0\pi^0)$. The S-wave interference contributions in the $D^0 \to \pi^0\pi^+\pi^-$ are negligible. One possibility for the discrepancy is the existence of other Dalitz plot solutions with relative phases closer to the flavor-SU(3) predictions. In spite of this apparent discrepancy in relative phases, however, the flavor SU(3) technique was able to reproduce branching fractions for the isospin ($I = 0, 1, 2$) amplitudes in $D^0 \to \pi^0\pi^+\pi^-$ [6] similar to those seen in experiments by BaBar [5]. This indicates that the flavor SU(3) technique is successful in capturing some of the essential physics.

The amplitudes and phases from the Dalitz plot for $D^0 \to \pi^0 K^+K^-$ are quoted in rows four through six of Table V. These results were discussed in more detail in Ref. [7]. Once again there is good agreement relative amplitudes between experiment and flavor SU(3), while relative phases don't agree so well. In this case we may set the phase for $A(K^{*+}K^-)$ to zero. This leads to a discrepancy in relative phases of about 18° for $A(K^{*-}K^+)$ and 57° for $A(\phi\pi^0)$. Note that the discrepancy is largest for $A(\phi\pi^0)$, for which we do not have a cross-ratio relation. In Ref. [7] we discussed several sources for these discrepancies

including inadequacy of the flavor-SU(3) approach, the possibility of having other Dalitz plot fits with relative phases closer to flavor-SU(3) predictions, and the need for proper parametrization of the relevant S-wave $K\pi$ amplitudes.

In the remaining three rows of Table V we calculate and compare flavor SU(3) predictions of amplitudes and phases from $D^0 \to K^-\pi^+\pi^0$ with experiment. As expected, the relative amplitudes from theory agree fairly well with experiment. Let us set the phase for $A(\rho^+ K^-)$ to zero to compare relative phases. We compare our results for relative phases with both the "3-resonance fit" as well as the "final fit" from Ref. [17]. We find that the relative phases obtained from flavor SU(3) for both $A(K^{*-}\pi^+)$ and $A(\overline{K}^{*0}\pi^0)$ agree with the "final fit" results and deviate from the "3-resonance fit" results by at most 2.6σ. In this case the flavor-SU(3) approach seems to produce results that are in acceptable agreement with experiment.

Finally in Table VI, where we compute and compare ratios of the amplitudes from Eq. (4) that relate these three Dalitz plots, we find that the amplitudes of these ratios agree very well with the results of the corresponding experiments. There is, however, a residual discrepancy in relative phases. Out of the three Dalitz plots considered for comparison this phase discrepancy is almost the same for $D^0 \to \pi^0 K^+ K^-$ (about 18°) and $D^0 \to \pi^0\pi^+\pi^-$ (about 17°.) In case of the $D^0 \to K^-\pi^+\pi^0$ Dalitz plot the relative phases agree with the "final fit" in Ref. [17] while there is less than 2σ deviation from the "3-resonance fit" result. The results of Ref. [17] are nearly ten years old. An updated analysis of the $D^0 \to K^-\pi^+\pi^0$ Dalitz plot could turn out to be useful in shedding more light on the effectiveness of the flavor-SU(3) technique.

VI Conclusion

We have shown that several ratios of amplitudes and phases for $D^0 \to PV$ decays are predicted to have the same value. The predictions of a flavor-symmetric SU(3) analysis for this universal ratio agree fairly well with results extracted from Dalitz plots for $D^0 \to \pi^+\pi^-\pi^0$, $D^0 \to \pi^0 K^+ K^-$, and $D^0 \to K^-\pi^+\pi^0$. Agreement is at least as good as that for the process $D^0 \to K_S\pi^+\pi^-$ compared previously with $D^0 \to \pi^0 K^+ K^-$ [7], with phase discrepancies limited to 20° or less. (Agreement with magnitudes of amplitudes is not surprising as the SU(3)-symmetric fits were based on observed branching fractions.) Cross ratios (independent of amplitude parametrizations) are shown to agree with predictions better than parametrization-dependent predictions such as the phase of the $D^0 \to \pi^0\phi$ amplitude. This could arise, for example, if the cross ratios were less affected by flavor-SU(3) breaking than other predictions of the flavor-SU(3) scheme.

The Dalitz plot analysis employed in our comparison for $D^0 \to K^-\pi^+\pi^0$ is nearly ten years old [17]. While it yields reasonably small errors in relative phases and magnitudes, considerably larger samples now exist, thanks to BaBar, Belle, and CLEO. It would be useful to update this analysis in light of the new data. Although S-wave $K\pi$ amplitudes play a relatively small role in this process, it would be good to explore various parametrizations of these amplitudes, as emphasized in Ref. [7], to be assured of the stability of the results.

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